PARTICULAR SOLUTIONS FOR THE GENERALIZED BURGERS-HUXLEY EQUATION BY THE FACTORIZATION METHOD

Taylanov Nizom Abdurazzakovich, Zokirov Mamajon,
Urazov Abduxoliq Nurmamatovich
Jizzakh State Pedagogical University, Jizzakh, Uzbekistan
e-mail: taylanov@yandex.ru

Abstract.*The article deals with numerical solutions of non-linear partial differential equations, such as the generalized Burgers-Huxley equations, which combine the effects of advection, diffusion, dispersion and non-linear transport. The Burgers-Huxley equation is solved numerically with the use of the factorization method, where a set of special solutions are obtained. It was shown that the factorization method is an efficient method with acceptable accuracy for solving the Burgers-Huxley equation.*
Keywords:*Burgers-Huxley equation, factorization method, nonlinear differential equations, modeling, traveling wave*

Faktorlashtirish usulida umumiylangan Burger-Huxley tenglama uchun alohida yechimlar
Аннотация.*Maqolada adveksiya, diffuziya, dispersiya va chiziqli bo'lmagan umumlashtirilgan Burgers-Xuksli tenglamalari differensial tenglamalarning sonli yechimlari ko'rib chiqilgan. Burgers-Xuksli tenglamasi faktorizatsiya usuli yordamida sonli usulda yechildi va maxsus echimlar to'plami olindi. Faktorizatsiya usuli Burgers-Xuksli tenglamasini echish uchun maqbul aniqlikka ega samarali usul ekanligi ko'rsatildi.*
Kalit so'zlar: *Burgers-Xuksli tenglamasi, faktorizatsiya usuli, chiziqli bo'lmagan differentsial tenglamalar, modellashtirish, harakatlanuvchi to'lqin*

Частные решения обобщенного уравнения Бюргерса-Хаксли методом факторизации
Аннотация.*В статье рассматриваются численные решения нелинейных дифференциальных уравнений в частных производных, таких как обобщенные уравнения Бюргерса-Хаксли, которые сочетают в себе эффекты адвекции, диффузии, дисперсии и нелинейного переноса. Уравнение Бюргерса-Хаксли решается численно с использованием метода факторизации, при котором получается набор специальных решений. Показано, что метод факторизации является эффективным методом с приемлемой точностью решения уравнения Бюргерса-Хаксли.*
Ключевые слова: *уравнение Бюргерса-Хаксли, метод факторизации, нелинейные дифференциальные уравнения, моделирование, бегущая волна*

Introduction

One of such equations is a non-linear Burgers-Huxley equation, which is applied to mathematical modeling of interaction between reaction-diffusion mechanisms, convection effects and diffusion transports [1], the problems of evolution of nonlinear waves in dissipative systems and several other nonlinear phenomenons like sound waves in viscous media [2]. Additionally, the Burgers-Huxley equation is widely used to model the dynamics of a range of physical phenomenon, for example, it is used to describe the dynamics of electric pulses that occur in nerve fibers and wall in liquid crystals [3]. Recently, much interest has been focused on the decomposition method.

This method refers to approximate methods for solving equations. In this paper, the efficient factorization scheme has been applied to generalized Burgers-Huxley equation. Particular solutions have been obtained for a number of important values of this nonlinear differential equation with applications in physics and biology sciences. The numerical implementation of the decomposition method was carried out in the Wolfram Mathematica computer algebra system.

Basic equations and results

The generalized Burgers–Huxley equation, which has the following form

$$u_t = u_{xx} - \alpha u^{\delta} u_x + \beta(1-u^{\delta})(u^{\delta}-\gamma), \qquad 0 \le x \le 1,\ t \ge 0, \tag{1}$$

$\alpha, \beta \ge 0$ and δ are the real constants and are positive integers. The variability of these parameters leads to many nonlinear evolution equations. Now, using the factorization method [4], we apply the traveling wave transformation $u(x,t) = U(\xi)$; where $\xi = k(x-ct)$ and k, c are the corresponding wave number and speed of the wave. Then, we obtain

$$\frac{d^2U}{d\xi^2} + g(U)\frac{dU}{d\xi} + F(U) + \frac{1}{k}(c-\alpha U^{\delta}) + \frac{\beta}{k}U(1-U^{\delta})(U^{\delta}-\gamma) = 0, \tag{2}$$

.

To continue this method, we factorize the function $F(U)$ and integrating the obtained equation gives a general solution of the form

$$U(\xi) = \left(1 \pm K\exp\left[-\frac{-\alpha \mp \sqrt{\alpha^2 + 4\beta(1+\delta)}}{2(1+\delta)k}\delta\xi\right]\right)^{-1/\delta}, \tag{3}$$

Next, we consider the specific solution the generalized Burgers–Huxley equation by numerical simulation. For instance, when α = 1, β = 2/3, δ = 1, and γ = 0, there is a rise in amplitude of u which results to a sharp gradient with flat boundary layer. Figure 1 shows the approximate solutions of the problem for several values of parameters as α = 1, β = 2/3, δ = 2, and γ = 0.

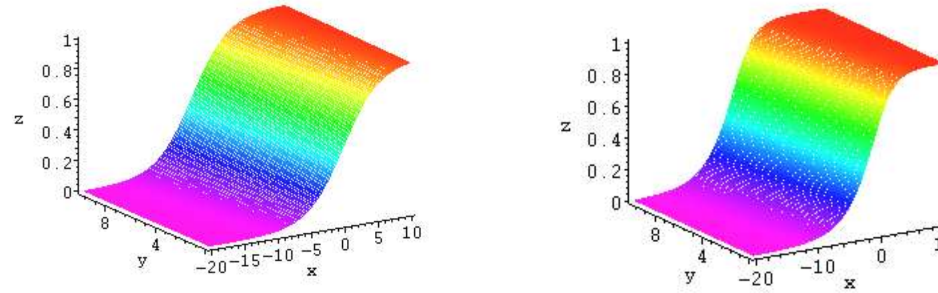

Fig.1. 3D plot showing the numerical solution of the generalized Burgers-Huxley equation for the case I at parameter values α = 1, β = 2/3, δ = 1, and γ = 0 at time level t = 0.02 for N = 200.

When α =2, β = 2/3, δ = 1, and γ = 0. Figure 2 shows the approximate solutions of the problem for several values of parameters as α = 2, β = 2/3, δ = 1, and γ = 0.

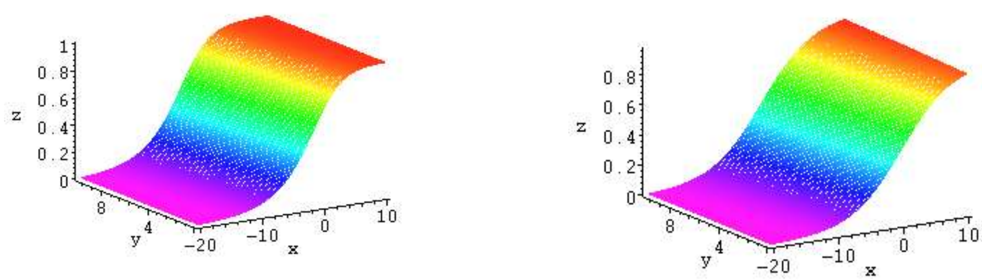

Fig.2. 3D plot showing the numerical solution of the generalized Burgers-Huxley equation for the case I at parameter values α = 2, β = 2/3, δ = 1, and γ = 0 at time level t = 0.02 for N = 200.

Now, consider the second case, when α =2, β = 2/3, δ = 2, and γ = 0. Figure 2 shows the approximate solutions of the problem for several values of parameters as α = 2, β = 2/3, δ = 2, and γ = 0.

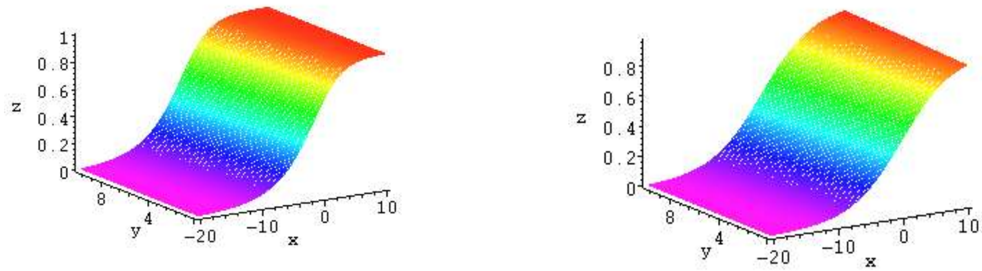

Fig.2. 3D plot showing the numerical solution of the generalized Burgers-Huxley equation for the case I at parameter values $\alpha = 2$, $\beta = 2/3$, $\delta = 2$, and $\gamma = 0$ at time level $t = 0.02$ for $N = 200$.

Conclusion

We have presented particular solutions for the generalized Burgers-Huxley in the form of traveling wave using the factorization method. It was shown that the factorization method is an efficient method with acceptable accuracy for solving the Burgers-Huxley equation. The obtained results prove that the factorization method is an efficient and powerful algorithm for constructing an exact solution to nonlinear differential equations.

References

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